

**MECHANICS 3 (A) TEST PAPER 8 : ANSWERS AND MARK SCHE**

1. (a)  $T \cos 60^\circ = mg$ , so  $T = 2mg$        $T \sin 60^\circ = m(L \sin 60^\circ)\omega^2$       M1 A1 M1 A1  
 Thus  $\frac{mg}{l}(L-l) = 2mg$        $L-l = 2l$        $L = 3l$       M1 A1
- (b)  $2mg = m(3l)\omega^2$        $\omega^2 = \frac{2g}{3l}$        $\omega = \sqrt{\frac{2g}{3l}}$       M1 A1      8
2. (a)  $mv \frac{dv}{dx} = -(mg + mf(x))$        $v \frac{dv}{dx} = -g - f(x)$       M1 A1  
 $v^2 = 2e^{-2gx} - 1$ , so  $2v \frac{dv}{dx} = -4ge^{-2gx}$        $-2ge^{-2gx} = -g - f(x)$       M1 A1  
 $f(x) = g(2e^{-2gx} - 1)$       (b)  $v = 0$  when  $2e^{-2gx} = 1$        $x = \frac{1}{2g} \ln 2$       A1; M1 A1
- (c) W.D. =  $m[-e^{-2gx} - gx]_0^{(\ln 2)/2g} = m(-e^{-\ln 2} - \frac{1}{2} \ln 2 + 1) = \frac{1}{2}m(1 - \ln 2)$       M1 A1 A1      10
3. Speed  $< u$ , so friction  $F$  acts up.       $R \cos \theta + F \sin \theta = mg$  (1)      B1 M1 A1  
 $R \sin \theta - F \cos \theta = \frac{mu^2}{4r}$  (2)      (1)  $\times \sin \theta$ , (2)  $\times \cos \theta$ , subtract :      M1 A1 A1 M1  
 $F = mg \sin \theta - \frac{mu^2}{4r} \cos \theta = mg \sin \theta - \frac{mg}{4} \tan \theta \cos \theta = \frac{3}{4}mg \sin \theta$       A1 A1 A1      10
4. E.P.E. changes from  $2 \frac{\lambda}{2l} l^2 (\sec 30^\circ - 1)^2$  to  $2 \frac{\lambda}{2l} l^2 (\sec 60^\circ - 1)^2$       M1 A1 A1  
 Gain in E.P.E. =  $\lambda l [(\sec 60^\circ - 1)^2 - (\sec 30^\circ - 1)^2] = \lambda l \left( \frac{4}{\sqrt{3}} - \frac{4}{3} \right)$       M1 A1  
 Loss in grav. P.E. =  $mg l (\tan 60^\circ - \tan 30^\circ) = \frac{2mg l}{\sqrt{3}}$       M1 A1  
 Hence  $\lambda l \left( \frac{12-4\sqrt{3}}{3\sqrt{3}} \right) = \frac{2mg l}{\sqrt{3}}$        $\lambda = \frac{3mg}{6-2\sqrt{3}}$       M1 A1 A1      10
5. (a) (i)  $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgr(1 - \cos \theta)$        $v^2 = u^2 + 2gr(1 - \cos \theta)$       M1 A1 A1  
 (ii)  $mg \cos \theta - X = \frac{mv^2}{r}$        $X = mg \cos \theta - \frac{mu^2}{r} - 2mg(1 - \cos \theta)$       M1 A1  
 $X = mg[3 \cos \theta - 2 - \frac{u^2}{gr}]$       A1
- (b) Leaves sphere when  $X = 0$ , i.e. when  $3 \cos \theta = 2 + \frac{u^2}{gr}$ , etc.      M1 A1 A1
- (c) If  $u^2 \geq gr$ ,  $P$  leaves the surface as soon as it is projected      B2      11
6. (a)  $mg = \frac{\lambda e}{l}$        $\lambda = \frac{mg l}{e}$       SHM:  $m\ddot{x} = mg - \frac{mg l}{el}(e+x)$       M1 A1  
 $\ddot{x} = -\omega^2 x = -\frac{g}{e}x$        $\omega^2 = \frac{g}{e}$       5 osc. per second, so      M1 A1 M1  
 $\omega^2 = 4\pi^2(5^2) = 100\pi^2$       Thus  $\frac{g}{e} = 100\pi^2$        $\frac{\lambda}{l} = 100\pi^2 m$       A1 M1 A1
- (b)  $mg = \frac{\lambda e}{l} = 100\pi^2 me$ , so  $e = \frac{g}{100\pi^2}$       M1 A1
- (c)  $T_1 = \frac{\lambda}{l} = \lambda = 100\pi^2 ml$  N      M1 A1      12
7. (a)  $\bar{x} \int_0^h \pi y^2 dx = \int_0^h \pi y^2 x dx$        $y = \frac{rx}{h}$ , so  $\bar{x} \int_0^h \frac{r^2 x^3}{h^3} dx = \int_0^h \frac{r^2 x^3}{h^2} dx$       M1 A1 B1 M1  
 $\bar{x} \frac{h^2}{3} = \frac{h^4}{4}$        $\bar{x} = \frac{3h}{4}$       M1 A1 A1
- (b) M(vertex):  $m \frac{2h}{3} + m \frac{3h}{4} = 2mkh$        $k = \frac{8h}{15}$       M1 A1 A1
- (c)  $\tan \alpha = r + \frac{7h}{15}$        $r = \frac{7h}{15} \tan 45^\circ = \frac{7h}{15}$  or  $0.47h$       M1 A1 M1 A1